1	(i) [radius = [centre] (4, 2)	**B1** **B1**	**B0** for ± 4	 condone omission of brackets

1	(ii) $(x-4)^2 + (-2)^2 = 16$ oe	**M1**	for subst $y = 0$ in circle eqn;	NB candidates may expand and rearrange eqn first, making errors – they can still earn this **M1** when they subst $y = 0$ in their circle eqn; condone omission of $(-2)^2$ for this first **M1** only; not for second and third **M1**s; do not allow substitution of $x = 0$ for any Ms in this part
	$(x-4)^2 = 12$ or $x^2 - 8x + 4$ [= 0]	**M1**	putting in form ready to solve by comp sq, or for rearrangement to zero; condone one error;	eg allow **M1** for $x^2 + 4 = 0$ [but this two-term quadratic is not eligible for 3rd **M1**];
	$x - 4 = \pm\sqrt{12}$ or $[x=]\dfrac{8 \pm \sqrt{8^2 - 4\times1\times4}}{2\times1}$	**M1**	for attempt at comp square or formula; dep on previous M2 earned and on three-term quadratic;	not more than two errors in formula / substitution; allow **M1** for $x - 4 = \sqrt{12}$; **M0** for just an attempt to factorise
	$[x=]4 \pm \sqrt{12}$ or $4 \pm 2\sqrt{3}$ or $\dfrac{8 \pm \sqrt{48}}{2}$ oe isw	**A1**		
	or	**or**		
	sketch showing centre (4, 2) and triangle with hyp 4 and ht 2	**M1**		
	$4^2 - 2^2 = 12$	**M1**	or the square root of this; implies previous M1 if no sketch seen;	
	$[x=]4 \pm \sqrt{12}$ oe	**A2**	**A1** for one solution	

1	(iii) t $\left(4+2\sqrt{2},\ 2+2\sqrt{2}\right)$ into circle eqn and showing at least one step in correct completion	**B1**	or showing sketch of centre C and A and using Pythag: $\left(2\sqrt{2}\right)^2+\left(2\sqrt{2}\right)^2=8+8=16$;	or subst the value for one coord in circle eqn and correctly working out the other as a possible value;
	Sketch of both tangents	**M1**		need not be ruled; must have negative gradients with tangents intended to be parallel and one touching above and to right of centre; mark intent to touch – allow just missing or just crossing circle twice; condone A not labelled
	grad tgt = −1 or −1/their grad CA	**M1**	allow ft after correct method seen for grad CA = $\dfrac{2+2\sqrt{2}-2}{4+2\sqrt{2}-4}$ oe (may be on/ near sketch);	allow ft from wrong centre found in (i);
	$y-(2+2\sqrt{2})=\text{their } m(x-(4+2\sqrt{2}))$	**M1**	or y = their $mx + c$ and subst of $\left(4+2\sqrt{2},\ 2+2\sqrt{2}\right)$;	for intent; condone lack of brackets for **M1**; independent of previous Ms; condone grad of CA used;
	$y=-x+6+4\sqrt{2}$ oe isw	**A1**	accept simplified equivs eg $x+y=6+4\sqrt{2}$;	**A0** if obtained as eqn of other tangent instead of the tangent at A (eg after omission of brackets);
	parallel tgt goes through $\left(4-2\sqrt{2},\ 2-2\sqrt{2}\right)$	**M1**	or ft wrong centre; may be shown on diagram; may be implied by correct equation for the tangent (allow ft their gradient);	no bod for just $y-2-2\sqrt{2}=-1(x-4-2\sqrt{2})$ without first seeing correct coordinates;
	eqn is $y=-x+6-4\sqrt{2}$ oe isw	**A1**	accept simplified equivs eg $x+y=6-4\sqrt{2}$	**A0** if this is given as eqn of the tangent at A instead of other tangent (eg after omission of brackets)

2 (i)	centre C' = (3, −2) radius 5	**1** **1**	 0 for ± 5 or −5
2 (ii)	showing $(6-3)^2 + (-6+2)^2 = 25$ showing that $\overrightarrow{AC'} = \overrightarrow{C'B} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ o.	**B1** **B2**	interim step needed **or B1** each for two of**:** showing midpoint of AB = (3, −2); showing B (0, 2) is on circle; showing AB = 10 **or B2** for showing midpoint of AB = (3, −2) and saying this is centre of circle **or B1** for finding eqn of AB as $y = -4/3\,x + 2$ o.e. and **B1** for finding one of its intersections with the circle is (0, 2) **or B1** for showing C′B = 5 and **B1** for showing AB = 10 or that AC′ and BC′ have the same gradient **or B1** for showing that AC′ and BC′ have the same gradient and B1 for showing that B (0, 2) is on the circle

2 (iii)	grad AC' or AB = $-4/3$ o.e. grad tgt = -1/their AC' grad $y - (-6) = \text{their } m(x - 6)$ o.e. $y = 0.75x - 10.5$ o.e. isw	**M1** **M1** **M1** **A1**	or ft from their C', must be evaluated may be seen in eqn for tgt; allow M2 for grad tgt = ¾ oe soi as first step or **M1** for y = their $m \times x + c$ then subst $(6, -6)$ eg **A1** for $4y = 3x - 42$ allow **B4** for correct equation www isw
2 (iv)	centre C is at $(12, -14)$ cao circle is $(x - 12)^2 + (y + 14)^2 = 100$	**B2** **B1**	B1 for each coord ft their C if at least one coord correct

3 (i)	10	**1**	
3 (ii)	[x =] 5 or ft their (i) ÷ 2 ht = 5[m] cao	**1** **1**	not necessarily ft from (i) eg they may start again with calculus to get $x = 5$
3 (iii)	$d = 7/2$ o.e. [y =] $1/5 \times 3.5 \times (10 - 3.5)$ o.e. or ft = 91/20 o.e. cao isw	**M1** **M1** **A1**	or ft their (ii) − 1.5 or their (i) ÷ 2 − 1.5 o. or $7 - 1/5 \times 3.5^2$ or ft or showing $y - 4 = 11/20$ o.e. cao

3 (iv)	$4.5 = 1/5 \times x(10 - x)$ o.e. $22.5 = x(10 - x)$ o.e. $2x^2 - 20x + 45$ [= 0] o.e. eg $x^2 - 10x + 22.5$ [=0] or $(x - 5)^2 = 2.5$ $[x =]\dfrac{20 \pm \sqrt{40}}{4}$ or $5 \pm \dfrac{1}{2}\sqrt{10}$ o. width = $\sqrt{10}$ o.e. eg $2\sqrt{2.5}$ cao	**M1** **M1** **A1** **M1** **A1**	 eg $4.5 = x(2 - 0.2x)$ etc cao; accept versions with fractional coefficients of x^2, isw or $x - 5 = [\pm]\sqrt{2.5}$ o.e.; ft their quadratic eqn provided at least M1 gained already; condone one error in formula or substitution; need not be simplified or be real accept simple equivalents only

4	i	(5, 2) $\sqrt{20}$ or $2\sqrt{5}$	1 1	0 for $\pm\sqrt{20}$ etc	2
	ii	no, since $\sqrt{20}$ < 5 or showing roots of $y^2 - 4y + 9 = 0$ o.e. are not real	2	or ft from their centre and radius M1 for attempt (no and mentioning $\sqrt{20}$ or 5) or sketch or solving by formula or comp sq $(-5)^2 + (y - 2)^2 = 20$ [condone one error] or SC1 for fully comparing distance from x axis with radius and saying yes	2
	iii	$y = 2x - 8$ or simplified alternative	2	M1 for $y - 2 = 2(x - 5)$ or ft from **(i)** or M1 for $y = 2x + c$ and subst their **(i)** or M1 for ans $y = 2x + k$, $k \neq 0$ or -8	2

	iv	$(x-5)^2 + (2x)^2 = 20$ o.e.	M1	subst $2x + 2$ for y [oe for x]	
		$5x^2 - 10x + 5[=0]$ or better equiv.	M1	expanding brackets and rearranging to 0; condone one error; dep on first M1	
		obtaining $x = 1$ (with no other roots) or showing roots equal	M1		
		one intersection [so tangent]	A1	o.e.; must be explicit; or showing line joining (1,4) to centre is perp to $y = 2x + 2$	
		(1, 4) cao	A1	allow $y = 4$	
		<u>alt method</u> $y - 2 = -½\,(x - 5)$ o.e. $2x + 2 - 2 = -½\,(x - 5)$ o.e. $x = 1$ $y = 4$ cao showing (1, 4) is on circle	 M1 M1 A1 A1 B1	line through centre perp to $y = 2x + 2$ dep; subst to find intn with $y = 2x + 2$ by subst in circle eqn or finding dist from centre = $\sqrt{20}$ [a similar method earns first M1 for eqn of diameter, 2nd M1 for intn of diameter and circle A1 each for x and y coords and last B1 for showing (1, 4) on line – award onlyA1 if (1, 4) and (9, 0) found without (1, 4) being identified as the soln]	
		<u>alt method</u> perp dist between $y = 2x - 8$ and $y = 2x + 2 = 10 \cos\theta$ where $\tan\theta = 2$ showing this is $\sqrt{20}$ so tgt $x = 5 - \sqrt{20}\sin\theta$ $x = 1$ (1, 4) cao	 M1 M1 M1 A1 A1	 or other valid method for obtaining x allow $y = 4$	 5 11